DBW-8 Seat No. ________

B. Sc. (Sem. II) (CBCS) (W.E.F. 2019) Examination

July - 2022

Mathematics : BSMT-02(A)

(Geometry, Calculus & Matrix Algebra) (New Course)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Numbers written to the right indicate full marks of the question.

1 (a) Answer the following questions briefly: **4**
(1) Write the equation of sphere in vector form.
(2) Find the sphere for which (1, –1, 1) and (–1, 1, 1) are the extremities of a diameter.
(3) Write the equation of cylinder whose axis is parallel to y-axis and radius r.
(4) Define : Right circular cylinder.

(b) Attempt any **one**: **2**
(1) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x + 6z - 6 = 0$.
(2) Write the equation of right circular cylinder with radius r and axis passing through origin.

(c) Attempt any **one**: **3**
(1) Find the co-ordinates of points where the line $\frac{x-1}{2} = \frac{y-6}{3} = \frac{z-4}{4}$ intersects the sphere $x^2 + y^2 + z^2 = 10$.
(2) Find equation of cylinder whose generator is parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and enveloping curve is $x^2 + y^2 + z^2 = a^2$.

(d) Attempt any **one**: **5**
(1) Find the equation of the tangent plane at any point (α, β, γ) of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.
(2) Derive equation of a cylinder of which generator remain parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through guiding curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0; z = 0$.

2 (a) Answer the following questions briefly: **4**

(1) Define homogeneous function of degree n in x and y.

(2) Evaluate $\lim_{x\to 0}\left[\lim_{y\to 0}\frac{\tan(x+y)}{x+y}\right]$.

(3) If $u = y^x$ then find $\frac{\partial u}{\partial y}$.

(4) If $u = \log\left(x^2+y^2\right)$ then find $\frac{\partial u}{\partial x}$.

(b) Attempt any **one**: **2**

(1) Discuss the existence of $\lim_{(x,\, y)\to(0,\, 0)}\frac{x^2-y^2}{x^2+y^2}$ by deriving iterated limits.

(2) If $f(x, y) = x\tan\frac{y}{x}$ then find $\frac{\partial f}{\partial x}$ at $(4, \pi)$.

(c) Attempt any **one**: **3**

(1) If $z = \sin^{-1}\sqrt{\frac{x^3+y^3}{x^2+y^2}}$ then show that

$$x\cdot\frac{\partial z}{\partial x}+y\cdot\frac{\partial z}{\partial y}=\frac{1}{2}\tan z.$$

(2) If $u = \phi\left(x^2+2yz,\, y^2+2zx\right)$ then prove that

$$\left(y^2-zx\right)\frac{\partial u}{\partial x}+\left(x^2-yz\right)\frac{\partial u}{\partial y}+\left(z^2-xy\right)\frac{\partial u}{\partial z}=0.$$

(d) Attempt any **one**: **5**

(1) State and prove "Euler's theorem" for homogeneous function of two variables.

(2) If $f(x, y) = 0$ then obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

3 (a) Answer the following questions briefly: **4**

(1) Define : Global maxima and minima.

(2) Define : Extreme point.

(3) If $x = p\cos\theta$, $y = p\sin\theta$ then $\frac{\partial(x, y)}{\partial(p, \theta)} =$ ________.

(4) $\frac{\partial(x, y)}{\partial(u, v)}\cdot\frac{\partial(u, v)}{\partial(x, y)}$ = ________.

(b) Attempt any **one**: **2**

(1) If there 1% error in the measurement of half of major and minor axis of an ellipse then evaluate relative error in measure of its area.

(2) For cylindrical co-ordinate $x = r\cos\theta, y = r\sin\theta,$ $z = z$ show that $J\left(\frac{x, y, z}{r, \theta, z}\right) = r$.

(c) Attempt any **one**: **3**

(1) If $f(x, y) = x^3 + xy + y^3$ then find the approximate value of $f(1.01, 2.98)$.

(2) If $u = x^2 - 2y, v = x + y + z, w = x - 2y + 3z$ then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 10x + 4$.

(d) Attempt any **one**: **5**

(1) State and prove Taylor's theorem.

(2) Find the greatest and smallest value of the function $f(x, y) = xy$ takes on ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

4 (a) Answer the following questions briefly: **4**

(1) Define : Rectangular matrix.

(2) Find trace of matrix $\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$.

(3) Give an example of 3×3 symmetric matrix.

(4) $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ then find $A \cdot B$.

(b) Attempt any **one**: **2**

(1) Prove that $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is Nilpotent of index 2.

(2) Prove that $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is Idempotent.

(c) Attempt any **one**: **3**

(1) If $AB = A$ and $BA = B$ then prove that A and B are Idempotent.

(2) Prove that A is involutory iff $(I + A)(I - A) = 0$.

(d) Attempt any **one**: **5**

(1) Prove that every square matrix can be expressed uniquely as the sum of symmetric matrix and skew symmetric matrix.

(2) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then find A^{-1}.

5 (a) Answer the following questions briefly: **4**

(1) Define : Characteristic equation.

(2) Define : Eigen values.

(3) Find the Eigen values of $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(4) Find the characteristics equation of $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

(b) Attempt any **one**: **2**

(1) Show that the matrices A and A^T have the same Eigen values.

(2) Prove that O is a characteristics root of a matrix iff the matrix is singular.

(c) Attempt any **one**: **3**

(1) Prove that matrix $A - \lambda I$ is singular iff λ is a Eigen value of matrix A.

(2) Show that the equation $x + y + z = -3, 3x + y - 2z = -2, 2x + 4y + 7z = 7$ are not constitent.

(d) Attempt any **one**: **5**

(1) State and prove Cayley-Hamilton theorem.

(2) Verify Cayley-Hamilton theorem for matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence obtain A^{-1}.